THE MATHEMATICS TEACHER

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JUDGING A TEACHER OF MATHEMATICS.

By WILLIAM E. BRECKENRIDGE.

(*Concluded from page 159.*)

(B) OUTSIDE OF THE CLASS ROOM the supervisor will note the general scholarship of the teacher as indicated by (*a*) his knowledge of subject matter, (*b*) knowledge of the history of the subject, (*c*) knowledge of its applications, and (*d*) the results of examinations. While examinations are not a reliable test of the quality of a teacher's work, yet every teacher is expected to pass a reasonable percentage of his pupils unless conditions are exceptional.

Another point to be considered is the *professional activity of the teacher* as shown by (*a*) periodicals taken, (*b*) interest and energy in the work of teachers' associations, (*c*) books and articles written for magazines.

A teacher should be active beyond his classroom work. What is his influence in school clubs and other points of contact with student life?

What is his power in the community as shown by church and social service? Finally, is he a *good executive?* Is he efficient in making reports and executing assigned work? Can he follow directions? Every school organization has more or less drudgery in the shape of reports and supervision that are necessary to the life of the school. A teacher should not consider himself above executing these duties, doing them on time, and in the precise manner required

TO HELP TEACHERS DISCOVER THEIR OWN STRENGTH AND WEAKNESS

TO HELP SUPERVISORS HELP TEACHERS WHERE NEED IS GREATEST

CLASSROOM INSTRUCTION CARD

SCHOOL	GRADE	H. S. YEAR	TOPIC	TESTS MADE IN

DATE	TIME : TO :	SUBJECT	PRESENT	REGISTER	CONFERENCE MIN.	OTHERS PRESENT

I—PERSONALITY OF TEACHER (Check √)

Illustrations

1 Teacher appears to be

vigorous	weak .....
poised	nervous .....
neat	slovenly .....
at ease	embarrassed .....

2 Voice is (Check √)

pleasing	harsh .....
clear	indistinct .....
low	high .....

3 Days absent on account of own illness since Sept. ..... to date ..... ;
during previous school year if teacher taught .....
Remarks:

4 In her personal relations with pupils does she appear (Check √)

to stimulate	to suppress	courteous	rude .....
to win cooperation ...	to antagonize ...	encouraging	nagging .....
to be sympathetic	harsh	firm	weak .....
strict	lax	tactful	blundering .....
even tempered	irritable	enthusiastic	diffident .....
reasonable	unreasonable ...	quick to react	slow to react .....
unduly serious	humorous	quiet	noisy .....
tolerant	intolerant	systematic	disorderly .....
dignified	undignified	resourceful	dependent .....

Illustrations

II—THE RECITATION (Underscore type of lesson) Drill, Problem, Teaching, Test

Illustrations

1 No. of pupils in class appearing

to be interested	indifferent .....
energetic	lazy .....
independent	dependent .....

2 Responses of pupils: No. giving

(a) fluent topical recitations .....
(b) word or phrase responses .....
(c) sentence responses .....
(d) incoherent responses .....
(e) failing to answer .....

3 No. of pupils in section

not reciting ..... reciting once ..... twice .....
three times ..... more than three times .....

4 No. of pupils asking

pertinent questions of fact .....
relevant thought-provoking questions .....

5 Time lost (Check √ under yes and no)

	Yes	No	No. Min.
(a) Calling class			
(b) Dismissing class			
(c) Distributing materials			
(d) Indistinct speech of teacher			
(e) Indistinct speech of pupils			
(f) Unnecessary talking of teacher, repeating pupil's answer			
(g) Unnecessary talking of pupils			
(h) Failure to have devices ready			
(i) Use of ill-adapted devices			

Illustrations

REVERSE SIDE OF CARD.

(Check √)

II—7 Teaching Ability as shown by	Not noted	Not at all	Slight	Medium	Notable	Evidence and Remarks
a EXTENT TO WHICH TEACHER'S QUESTIONS ARE						
(1) thought provoking						
(2) calling for facts						
(3) suggesting the answer						
(4) answered by "yes" or "no"						
(5) irrelevant						
(6) not definite—vague						
b EXTENT TO WHICH MATERIAL TO RECITATION IS						
(1) confined to text						
(2) within pupil's comprehension						
(3) related to children's lives and experiences						
(4) adapted to children's present or future needs						
(5) worth while						
c EXTENT TO WHICH THE TEACHING						
(1) is rambling						
(2) is formal, mechanical						
(3) stimulates initiative of pupils						
(4) requires independent thinking						
(5) develops pupil's resourcefulness						
(6) requires cooperation of pupil						
(7) is fixed on essentials						
(8) requires pupils to organize material						
(9) utilizes children's experience						
(10) clears up pupils' difficulties						
(11) shows use of material in solution of present or future problems						

(Check √)

Teaching Ability	N. N	N.	S.	M.	N.	Evidence and Remarks
d EXTENT TO WHICH PUPILS						
(1) had a clear idea of purposes of lesson						
(2) were self-reliant						
(3) tested their own solutions						
(4) acted and thought on their own account						
(5) cooperated with teacher and classmates						
(6) persisted in getting desired result						
(7) differentiated between essentials and non-essentials						
(8) organized their material						
(9) seemed well-grounded in previous work						

8 Was the Assignment	Yes	No
a. definite and clear?		
b. related to present lesson?		
c. such that the pupils were prepared to attack it intelligently?		
d. formal—from text book?		
e. by topics or problems?		
f. hastily made at dismissal?		
g. omitted?		

9 Correction of Essential Errors

(1) Describe method used .....

(2) Are non-essential errors too much emphasized? .....

(3) What record is kept of recurring errors likely to retard progress of pupils? .....

Distractions for pupils or teacher	Temperature ..... oppressive .....
noise in halls ..... storm	Bad air ..... fresh air .....
noise in street ..... interruptions	Teacher made adjustments ..... did nothing .....

..... Examiner

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If you say that a teacher who could measure up to all of these points should immediately be translated to the world of perfect beings, you state a truth. But this is the limit which our efficiency should indefinitely approach. You will also say that a supervisor cannot keep all of these points in mind. This also is true. It is recommended that he make out a brief list of those which he considers important.

Or he may use a card with all of these qualities listed and mark only those which seem most important in the case of each teacher. As a suggestion of how a summary of points could be arranged a card is appended published by the Bureau of Municipal Research, 261 Broadway, N. Y. City. This may also suggest to supervisors certain convenient adjectives for use in writing recommendations for teachers and in making out reports on teachers' efficiency.

Additional material on the topic may be found in the Teachers' Year Book No. 2 published by the Division of Reference and Research, 500 Park Ave., N. Y. City. In this book are found plans for judging a teacher, assigning quantitative values to the various points as well as schemes for qualitative analysis like that on the appended card.

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RIGHT AND WRONG DEFINITIONS OF A LIMIT.

By Edward V. Huntington.

The following comparison of the correct definition of the *limit of a sequence* with three other definitions, which, although incorrect, are still occasionally to be met with in elementary textbooks, may be of interest to teachers who wish to clarify their ideas not only as to what a limit is, but also as to what it is not.

The six illustrative examples will serve to show that the logical content of each of the four definitions is really different from that of each of the other three.

Definition 1. (Correct.) Suppose we have a sequence of values, $u_1, u_2, u_3, \ldots$, progressing according to any given law; and also a constant quantity c. Then the constant c is called the *limit* of the sequence, *provided* whatever quantity k your opponent may select, *you can always find a stage in the sequence* such that for all values of u beyond this stage, the difference between u and c is less than k.

In other words, whenever your opponent selects a value of k at pleasure, you must be able to find a point in the sequence such that all the values of u beyond this point lie within the range $c-k$ to $c+k$. If, for any selected value of k, the corresponding point in the sequence *cannot* thus be found, then c is *not* the limit of the sequence.

The same definition can be expressed more briefly as follows:

A constant c is called the limit of a variable u, if the difference between the constant and the variable eventually becomes and remains smaller than any pre-assigned quantity k.

Definition 2. (Wrong.) A constant c is called the "limit" of a variable u, if every change in the value of u brings it nearer to c.

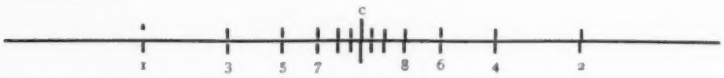
Definition 3. (Wrong.) A constant c is the "limit" of a variable u if the difference between c and u can be made less than any pre-assigned quantity, however small.

Definition 4. (Wrong.) A variable u is said to have the

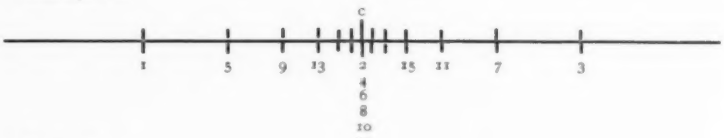
"limit" c, if it continually approaches nearer and nearer to c, but never reaches c.

In each of the following illustrative examples, the successive values u_1, u_2 u_3, etc., are represented by points along a line, or rather by the distances to these points from a fixed origin O. The law of variation will in each case be sufficiently clear from the figure, the variable being supposed to run through the values marked 1, 2, 3, etc., in order.

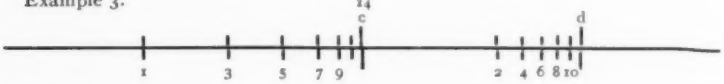
Example 1.



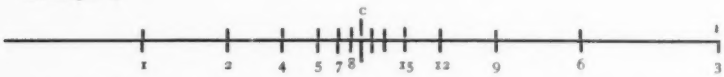
Example 2.



Example 3.

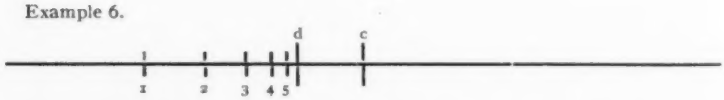


Example 4.



Example 5.

Example 6.



If now we inquire in each of these cases whether the constant c is the "limit" of the variable u, according to each of the four definitions, we obtain the results exhibited in the following table:

Def.	1	2	3	4	5	6
I.....	Yes	Yes	No	Yes	Yes	No
II.....	No	No	No	No	Yes	Yes
III.....	Yes	Yes	Yes	Yes	Yes	No
IV.....	Yes	No	No	No	No	Yes

It is clear from the definitions that every instance which is "yes" under definition I. will also be "yes" under definition III. With this necessary exception, the table shows that whatever two of the definitions we choose to compare, there is always an instance which is "yes" for one and "no" for the other, and also an instance which is "no" for the first and "yes" for the second. In other words, the four notions of "limit" embodied in the four definitions are absolutely distinct from one another, and should never be confused.

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MATHEMATICS AND PSYCHOLOGY.*

By C. C. Grove.

In mental vision, upon the wall of a great hall, I see a clock with weights and pendulum exposed. An electric light is dependent before it, causing dark clear shadows on the wall—of the weights like inverted pine-cones, of the pendulum like a wicked one fleeing with none pursuing, faster and still faster, ever fainter, to the other end of the hall.

It often happens in human affairs that men get their attention focused on the scurrying shadow and not on the stately rhythmic swing of the pendulum, upon the unreal instead of the real. Thus it was, several years ago, that a friend, an able teacher of secondary mathematics, came asking about certain declarations then recently made about the value and future of mathematics in the secondary schools. There was real concern in his questioning. Instinctively I thought of one Demetrius, the silversmith, of Ephesus.

At that time, as now also, I thought there was no occasion for concern to those who are of the way, to those who have breathed in the spirit of mathematics, whose vision is clear to see the true normal swing of the pendulum. I further believe now, as then, that the permanent appreciation and conception of our subject on the part of the masses rests with, and shall remain in the hands of those who teach mathematics.

It is for us to examine ourselves to see whether we have re-

*Under the above broad title, so stated for brevity on the program of the annual meeting of the Association at Drexel Institute, November 27, 1915, there was presented a paper that dealt with two separate lines of contact of the departments of psychology and mathematics. The first part considered an attitude towards the so-called "movement against mathematics in the secondary schools" which was in large part initiated on account of conclusions drawn from investigations made by experimental psychologists. This part follows. The second part, to be published in the near future, deals with the mathematics employed by psychologists, and points out the foundations on which this mathematical work rests.

ceived the spirit of mathematics. If we have not, we shall be troubled by many things; if we have, we shall not be alarmed by the statements of those who have not. Until we recognize the spirit of mathematics as the sun about which the whole system revolves, a hundred questions will harass and perplex us—questions as to transfer, as to parts of that formal outer husk and hard shell of mathematics—the mere formal processes and rules—that will transfer and be useful in the curricula of the schools of various types. Too much teaching of mathematics has been a "pouring-in process" and a drill in formal operations, instead of a presentation of the method of inquiry into any subject and the means of making intelligible a large body of the world's intellectual activity. Mathematics is not "simply deductive logic" but claims sisterhood with the humanities and is quite at home in their company. Not all interpretative courses need be confined to history and philosophy. Read addresses by Professor Cassius J. Keyser in SCIENCE, Nov. 12, '15, *The Human Significance of Mathematics*, and Mar. 26, '15, *Mathematical Instruction*.

Teachers who get no further in their understanding and appreciation of mathematics than the formal stage, just indicated, are in Milton's words "blind mouths" whose

> lean and flashy songs
> Grate on their scrannel pipes of wretched straw,

while

> The hungry sheep look up and are not fed.

They are neither bishops nor shepherds. Rather we look to the teacher who feels with Browning the vernal throb of new abounding life and says,

> Grow old along with me!
> The best is yet to be,
> The last of life, for which the first was made.

Should not the heart beat once, "How good to live and learn"? These teachers have spiritual children. The spiritually dead cannot beget spiritual children. The letter killeth, the spirit giveth life.

We must recognize mathematics as "the ideal and norm of all

careful thinking" (G. Stanley Hall), that only through mathematical reasoning can we approach the ultimate reality of the eternal world, for, as Bergson has said, "the more science digs into things the more it tends to mathematics." Yet we must also see that the spirit and method of mathematics pervade the humblest human activities.

The maid asked my wife, "Does Mr. Grove wish to alarm the clock?" I have seen a person turn a profile photograph edgewise to get the full-face view of the friend represented. I have heard a person, telling of an unsuccessful hunt for a certain number on a street in Philadelphia, say, "One went on each side of the street and we walked for blocks without finding it." Someone said "Poor horse, one would think he could not fall with all that harness on." You laugh, yet all these except the first were spoken by educated people. Quietly examine how many people act and speak as though they were the children of Epimetheus instead of Prometheus; how many cannot follow directions, read a recipe; how many attempting to do something, miss the very heart and purpose, do the manifest external processes, and so leave it half done, which is frequently worse than not done at all. Observe how difficult it is for a physician to get the essentials to diagnose a case. Many looking for peas cannot see pumpkins all around. Thus we might multiply illustrations of how people very generally do not analyze the situations of life and do not act intelligently. Dr. Katherine B. Davis, Commissioner of Corrections of New York, in an address said, "If only the schools would teach children to appreciate and distinguish cause and effect, they would do more than by teaching geography, history, etc."

Now it seems highly reasonable that a course that will present to children conditions clearly stated and will demand careful accurate thinking on the same, with a clear correct statement in good English of the analysis of the conditions and the logical result, will furnish a drill, if kept within the capacity and experience of the child, that will not permit such loose superficial thinking, or lack of thinking. Such a course, in my experience, is a properly administered course in mental arithmetic. One of the first fruits of the Pestalozzian movement and the beginning of true reform in mathematical teaching in the United

States was Warren Colburn's "Intellectual Arithmetic" (1821). I received an incalculable blessing by such a course from Brooks's "Mental Arithmetic." Into some such courses there came to be introduced so many special features that the real issue was overlooked. Even where this was not the case, teachers often found it very difficult to conduct the course properly because they had not caught the spirit of it. Then it was easy to say, "We are spending too much time on arithmetic." The statement in itself was very true but the remedy altogether wrong; they discontinued mental arithmetic and continued the formal "written arithmetic" under teachers often that had to have the "Key" as their indispensable "vade mecum."

Experience tells me that some such careful training in analysis, coupled with the demand for accurate statement in good English, is an essential, alike for men and women, for the intelligent conduct of life in any sphere. The habit of analyzing problems of life, of probing to the real issues, of demanding sound not merely plausible reasons, the habit of thinking and not seeking intellectual salvation without it—these, I say, are fostered directly by such study and the question of transfer does not arise, for the process of thinking is identical in each. The conditions in mathematical problems are usually clearly and fully given; in life the conditions must be determined by experience and a form of scientific imagination. The reasoning is the same however. Thinking is thinking; and the great majority of people try to avoid more than is absolutely necessary of it because they have not been trained to it nor learned the joyful mastery it brings to those who have the habit. This habit is not to be formed by the study of mathematics alone; yet through such study the habit, we feel, can be most readily and thoroughly formed, because here possibly better than in any other study can be demanded "the patience to be thorough, the concentration to understand, and the persistence to grasp and to apply" (President Butler's Annual Report, 1913, p. 35), and failure can be most easily detected.

[In these and other words, in the oral delivery of the paper, I tried to show that the chief concern of mathematicians as to "the movement against mathematics in the schools" need be

only to continue quietly and more earnestly to lead the increasing number of good students into a realization of the *spirit* of mathematics, to train teachers who, with due acknowledgments to Robert Browning, regard what they have to offer and present, "not merely as a substitute for a good cigar or a game of cards," nor yet as a great spoonful of the treacle of Squeers, that must be swallowed willy-nilly. Such teachers will consider that they fail of their mission unless they teach *more* than mathematics, unless mathematics becomes for them the best means of inspiring students with a love for truth and with a joy in seeing a body of truth being built up into a great cathedral-like structure as the embodiment and expression of the life and character of the builder. Such teaching is its own motivation for student and teacher alike. Like the thoroughbred, it has the spur in its veins. If you do not believe it, try it or go sit under a teacher who will demonstrate it to you.

Then were pointed out some sorts of the barnacles encrusting our educational bark:

The large proportion of young unmarried teachers who have no idea of following teaching as a profession, as recently shown by Mr. Harold W. Foght, of the U. S. Bureau of Education, Bulletin 49, 1914. These naturally lower the spirit of teaching for they bring little or no spirit to it.

A certain class of educators who, considering the proposition that there is no transfer of mathematical ability to have been proven by psychologists, are doing much harm by loudly advocating a "practical vocational" training, which turns out undisciplined, uncultured artisans, whose "primary object" is to have "good producing ability as a marketable asset," and also by getting the child to elect to place himself into a veritable caste system before he has seen any more of the glory of living than they choose to see. They do not recognize that at any time the industrial equilibrium may be upset and it may be necessary to bring the same body of workers, through long costly training, into successful readjustment.

Also the all too common practice of teaching down to children, as opposed to letting them come to enjoy the fellowship and companionship of their elders. Look up the life of John Ruskin, of Oscar Wilde, of Carl Witte, of Horace Mann, and

read the memorial to the late Dr. James M. Greenwood in *School and Society* of March 20, 1915, pp. 409, 410. Parents and teachers alike have much to learn of the inestimable value of the intimate friendly association of young and old.

Another form of teaching down to children is repeated drilling on what the child comprehends at the first presentation, until the child fairly groans within himself to be freed from that dead body of truth, loosed from the hold of the whirlpool into which he has come.

Such factors influencing education throughout, barnacles retarding its progress, are not recognized, their effects are charged to the *subject* of mathematics instead of to the spirit in which it is taught, and the ground for criticism is apparently much increased.

Finally there has been strongly claimed for the conclusions drawn from so-called mental tests that these rest on a sure scientific, mathematical basis. Many, not knowing statistical methods, have then transferred to the interpretations of the results of these tests the universal feeling of certitude or confidence in mathematics. Here, you will note, is a case of actual transfer far more easily proven than that mathematical training does not transfer. May it not be that the contention is that of the fly as to the elephant in the introduction to the "Budget of Paradoxes," and that all that is needed is to await the coming or birth of more flies when the matter will settle itself? In fact it seems that many without the psychological ability, insight and experience of their master may bring even the good accomplished into disrepute and doubt by rushing into formal applications of his methods. The great increase in carefulness in expression in recent years may be seen by comparing, *e. g.*, Professor E. L. Thorndike's "Educational Psychology," 1913, Vol. II, p. 425, with earlier statements running back to 1903.

Thus we come to our initial statement that the alarmists have their attention on the scurrying shadow and not on the real situation. We want to get, keep, or continue disseminating the spirit of mathematics according to our individual positions. We need to be brought by the criticism to serious self-examination of ourselves and our teaching, and to bring the latter to its

proper, high plane. The criticism will then, like a summer's cloud, soon disappear.

The psychologists, I know, are glad to learn where they have gone astray, and especially to learn the fundamental assumptions made in the development of the statistical methods they have employed; also the applicability or the non-applicability of the methods to their problems. This you will find in the forthcoming second part of the paper.

(*To be concluded in next issue.*)

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"TEACHERS' MARKS."

By C. S. Bragdon.

The chairman of the program committee suggested as an appropriate topic for this occasion a review of some recent book or books on educational topics of interest to mathematics teachers. As I was browsing among the newer books on education in our library a little volume attracted my attention. It was entitled "Teachers' Marks"—by Frederick James Kelly, Ph.D. Most teachers consider the marking of papers a bore and the least interesting phase of a teacher's work. What could be more tedious, therefore, than a scientific treatise, composed of tables of figures and per cents. and charts on this subject? But just as nuggets of gold are sometimes found in unexpected places so this seemingly unattractive book was found to contain material of unusual interest. Because it interested me, I concluded that it might not prove unattractive to you teachers of mathematics who are supposed to revel in figures and statistics.

I shall not attempt a review of the book, but allow it to supply us certain facts and fiugres which will form the basis of a brief discussion of the topic suggested in the title.

Passing over the first chapter on grading the work in elementary schools to the second which treats of standards of marking in high schools, Mr. Kelly quotes from a study made by Mr. F. W. Johnson of the University High School of Chicago figures to show the wide divergence of marks given by two teachers in the same subject in that school. The first teacher had 8 per cent. failures and 7.5 per cent. A's or highest grade; the second had 4.5 per cent. failures and 36 per cent. A's. When teachers in different subjects were compared a still greater divergence in marks was found; one teacher having 26.5 per cent. failures and 1.5 per cent. A's as compared with 4.5 per cent. failures and 36 per cent. A's for another teacher.

That the passing standard is largely a matter of tradition or whim is the conclusion the author draws from the statistics of a

certain New York City high school. A change of principals was made in this school in 1910. In the previous year only 48 per cent. of the algebra pupils were passed the first term and 61 per cent. the second term, while the new principal decided to pass 75 per cent. of the algebra pupils the first term and 80 per cent. the second.

A series of tables then follows showing the successive marks received by pupils from grade to grade, through high school and college. The discovery is made that only about 50 per cent. of the large number of cases studied retain the same relative position by thirds or tertiles, *i. e.*, of those in the upper third of the elementary grades only about one half maintained a place in the upper third of their classes in high school, etc. From this study the author draws the following conclusion: "If we can come no nearer than that in ranking our children for general ability, we cannot hope to command much respect as a teaching profession. Rather should the revelations made by these studies open our eyes to the real need for some more effectual method for establishing standards whereby both teachers and pupils may measure progress."

Passing next to the marking of examination papers, he says: "The few studies which have been made reveal a very wide difference of rating upon the same papers among supposedly competent judges." In support of this contention he cites the experiment made by an Oxford professor who caused to be inserted in the *English Journal of Education* a specimen of Latin prose composition. He then invited competent judges to rate the paper and send him their results. Twenty-eight replies were received with the marks as follows: 45, 59, 67, 67.5, 70, 70, 72.5, 6 at 75, 77, 5 at 80, 2 at 82, 2 at 85, 87.5, 88, 90, 2 at 100.

The results of an experiment at Teachers' College, Columbia University, also confirm the above statement. Eleven judges, all graduate students, were asked to rate a set of twenty papers in geography. The marks on two of these papers will be sufficient to illustrate the diversity of ratings. Paper No. 4 was rated by the eleven judges as follows: 27, 50, 60, 28, 59, 60, 48, 40, 90, 72, 15, 50. Paper No. 17—53, 50, 90, 54, 93, 63, 46, 60, 100, 39, 100, 59.

At the University of Wisconsin the following experiment was

tried: A facsimile reproduction was made of a geometry paper written by a pupil in one of the leading high schools of the state. Copies were then sent to a large number of schools with the request that the paper be rated by the teacher in each school best qualified for the work. The 116 replies contained marks as follows: 1 at 28, 1 at 39, 1 at 41, 1 at 44, 2 at 48, 6 at 50, 6 at 54, 8 at 55, 8 at 59, 17 at 60, 17 at 64, 19 at 65, 19 at 69, 7 at 85, 7 at 89, 2 at 90, 2 at 94.

I will quote but one other experiment in marking of a set of papers in mathematics from an Indiana high school. The regular teacher had given to this set an average mark of 78.7 per cent. Five other competent persons were asked to rate the same papers. One gave to the set an average mark of 74 per cent., the second, 61.4 per cent., the third, 65.5 per cent., the fourth, 75.5 per cent., the fifth, 58.0 per cent.

Mr. Kelly then says: "In all of the above studies we see very serious lack of standards among teachers. It is true that in all these cases the judges were selected from an area where no especial effort had been made to standardize the judgments. On this account, I undertook to measure the variations between the marks of teachers in New York State on the one hand and of the regents on the other." He then quotes statistics from the regents' reports for several years up to 1913 and says: "The two tendencies to which attention is called are the constantly increasing per cent. of papers which the regents have passed and, at the same time, the constantly increasing per cent. of papers rejected by the regents of those passed by the teachers. These two tendencies seem to me significant. While an ever increasing number of pupils in the high schools of the state are able to meet the requirements of the examiners, the difference in the standards of judging papers by teachers and examiners grows ever greater. While the requirements for high-school teachers are constantly being increased, their judgment of the value of examination papers is being more and more rejected. The greater the per cent. failed by the teachers, the greater the additional per cent. failed by the regents. The rule is not even violated in the case of mathematics which by all tradition offers the greatest possibility of exactness in marking papers." Continuing, he says: "It is a certain indication that there is as little

agreement among the teachers of the state concerning standards hoped for by the regents themselves in the examinations as there is among the examiners in the various subjects."

He then asks a question concerning the fate of papers rated by the teachers around 60 per cent. and finds these figures to be true: "41.3 per cent. of such papers were failed by the regents, 5.64 per cent. raised above 65 per cent. and more than one half left at 60 per cent." "The chief interest," says he, "in this table lies in the report common among high-school teachers that they push up the grade on doubtful papers to take a chance on their passing. Results seem to indicate that the policy is a wise one, for of papers marked at 60 per cent. the lowest per cent. saved to any school was 22 per cent., the highest 75 per cent., the average 58.7 per cent. Finally, by all these findings concerning the New York state system of examinations we are compelled to conclude that the type of examination now in common use is *not* a successful means of standardizing school achievement."

The writer then proceeds to suggest a better means of standardizing the results of examinations, as shown by an experiment in the schools of Orange, N. J. A uniform test was given to all fifth-grade pupils in arithmetic. These papers were rated by the respective teachers. Afterwards a most efficient and sympathetic teacher was asked to rate all the papers and compare her results with the results of the others. Afterwards this teacher was asked to submit an appropriate scheme for marking each question on the test. Then the first teachers were asked to re-rate their papers using this scheme. The results showed a very considerable range of difference when teachers used their own standards of marking, but when the same scheme is used by both teachers and judge the range of differences is very much reduced, considerably more than half the cases being zero. "From this experiment," he says, "we may draw one lesson: If the superintendent expects to place much significance upon the uniform tests he gives he must either have the marking done by a single judge, or else he must make out a scale for the rating of the papers by which the variations of the several teachers may be greatly reduced."

In the remaining fifty pages the author examines the various

devices for standardizing results, the most interesting ones to teachers of mathematics being those of Stone and Courtis pertaining to arithmetic. No discussion of them, however, will be attempted in this paper, as I believe it will be more profitable to spend the remainder of our time in considering some of the statements and criticisms already quoted.

We may summarize the conclusions of Mr. Kelly under the following heads:

I. There is wide variation in rating work of a definite grade, due to the point of view or personal equation of the teacher.

II. Such wide variation constitutes a serious reflection on the standing of teaching as a profession, since those supposed to be experts vary in marking the same paper between 40 per cent. and 100 per cent.

III. The examination as a means of measuring the proficiency of pupils and furnishing a basis of promotion is of little value unless a method of standardizing results may be devised.

IV. The regents system of uniform examinations is failing to provide the desired standardization as shown by the great differences between the judgment of the teachers and that of the examiners of the state department.

The first three conclusions will readily be granted. The fourth, concerning the failure of the regents system to secure uniformly reliable results, contains much truth. The author, however, neglects to state the fact that the examinations division urges a system of marking which, if generally followed, would assist greatly in securing the desired results. I refer to the committee system of marking with which you are undoubtedly familiar. To be sure, more time is required to rate a set of papers by this method, but the results are enough better to warrant the additional time and effort. It does seem, however, that after three competent teachers have agreed upon a scale of marking and then have carefully rated a set of papers by the committee system that their combined judgment of the marks to be given must be as nearly accurate as it is possible to obtain. Why, then, the need of having such papers re-rated at Albany? Does it not mean the setting aside of the judgment of a committee of competent examiners and substituting therefor the judgment of an individual no more competent than any one of the

committee? Do not the statistics quoted from Mr. Kelly's book prove that the judgment of an individual, no matter how competent, is less likely to be correct than the combined judgment of several? Would it not therefore be safe for the examiners to accept at face value, without re-examination, all papers which have been carefully marked by the committee system? Would there not be a very strong inducement for more schools to adopt the committee scheme of marking if they knew that their marks would be accepted. If the combined judgment of three teachers claims a paper at 64 and then a single examiner rejects it at 58 what inducement is there to schools to mark by the committee system?

Not long ago I received a letter from a high-school principal in which he said: "We would like to rate our papers about as they will be rated at Albany, but find it quite impossible. Are you having the same experience?" It seems to me that this little book by Mr. Kelly proves that agreement between the schools and the examinations division is impossible so long as the mark of an individual teacher in any school is compared with that of an individual examiner at Albany. This statement is in no sense a reflection on the judgment of the teacher nor on that of the examiner. It is simply the inevitable consequence of the difference in points of view of two equally competent judges working independently without a prearranged standard of marking. Failure to appreciate the full significance of this great truth frequently leads to mutual distrust of the examiners by the schools and of the schools by the examiners. Such a condition ought not to exist and would not if a closer understanding between teachers and examiners could be effected. The sentiments expressed by the high-school principal quoted above are shared by all principals. We would all like to have our teachers rate their papers as the examiners wish them to be rated. And right here I wish to protest against a current notion that we are concerned merely with the number of papers accepted or rejected. We are just as anxious, yes, even more anxious to know why a paper rated by our teachers at 93 is reported back with a mark of 81 as to know why a paper just on the border line of failure is rejected. Especially is this true since such a reduction of ten or more points may mean for a pupil the loss of a state

scholarship. The privilege of reviewing high papers thus reduced in mark ought to be used much more than is now the case, because by so doing a better mutual understanding of systems of marking may be realized. The real purpose of the examinations is not to determine how many pass or fail, but to give a just and reasonably accurate rating to all papers, high or low. And teachers and examiners must come together on this matter and agree on standards understood by all so that such differences in rating may be avoided.

But some may say that such standards have been established by the specialists in the various subjects. Quite recently a new series of pamphlets entitled "Suggestions on the Rating of Regents Examinations" has been forwarded to the schools. We all hoped that these new suggestions would be sufficiently explicit and specific to constitute a standard for rating papers, and thus eliminate largely the possibility of widely divergent marks for papers of equal merit. But a careful reading of these suggestions shows them to be disappointing in this respect. In the suggestions for rating mathematics papers several statements are made which are capable of widely differing interpretations, so that we are just as far from a definite standard by which to rate our mathematics papers as we have been in the past. Until such a definite standard has been adopted our uniform examinations can never bring uniform justice to our schools nor to the pupils in our schools.

That the examinations division appreciate the truth of this assertion is evidenced from the following statement from Mr. Horner, chief of the examinations division, in his report for 1913. He says: "Something more than suggestions is needed to secure a fair degree of uniformity in rating answers." He then advocates the use of the committee system to which I have referred as a means of preventing serious differences in rating that are due to differences in temperament. In the same report Mr. Horner says: "It is hardly too much to hope that registered secondary schools may some time be brought to such a high standard that the local ratings will be final in all cases." Surely the examinations division is not more anxious to see this hope realized than are the principals and teachers of our secondary schools, yet according to the 1914 report only 57 schools out of nearly 900 had 95 per cent. or more of its papers claimed ac-

cepted. In other words, less than 7 per cent. of the secondary schools of the state seemed to have a sufficiently clear comprehension of the standard of the examinations division to have 95 per cent. of its claimed papers accepted, while in 150 secondary schools not more than 70 per cent. and in some schools as low as 45 per cent. of its papers claimed were accepted.

These figures force us again to the conclusion that some means should be found for more clearly defining the standard of the examinations division, for if that standard were clearly understood at least 50 per cent. of the schools of the state ought to be able to rate their papers by the committee system in harmony with the standard. What a saving of time and money this would mean to the state for, to quote Mr. Horner again, "It is worse than useless for the department examiners to spend their time in rereading thousands of papers rated by teachers fully as competent and as accurate in rating as they are."

In closing may I venture to suggest another possible means of securing greater uniformity in rating regents' papers. The examinations in mathematics, for example, are compiled by a committee of experienced teachers acting with the specialist in mathematics of the department. What body could be more competent than this to formulate a definite scale for marking the various parts or questions on these papers? Specific directions for marking each mathematics examination could thus be formulated by this committee; these directions could be printed and a few copies sent to each school with the question papers. Would not such a set of specific directions for marking a particular examination tend to secure far greater uniformity in marks than a set of general suggestions intended for several different types of examinations? Possibly some teachers might object to such a plan on the plea that it would deprive them of their individuality and make them mere marking machines. But if uniformity in marking is desirable, then the personality of the marker must become of less relative importance than conformity to the standard. For the marks should show the true relative worth of the work of an individual pupil in comparison with the work of all others taking the same examination. In this way only can justice be secured.

UTICA FREE ACADEMY,
UTICA, N. Y.

REPORT OF COMMITTEE OF THE NEW ENGLAND ASSOCIATION OF TEACHERS OF MATHEMATICS ON SECONDARY SCHOOL MATHEMATICS, APRIL, 1916.

TABLE OF CONTENTS.

I. INTRODUCTION.

The committee was appointed May 9, 1914, "to consider what action or attitude could wisely be taken by the Association with reference to more or less prevalent, and more or less responsible,

criticism of secondary school mathematics." Twelve meetings have been held.*

The announcement of the committee's appointment elicited an interesting letter from Dr. David Snedden—then Commissioner of Education in Massachusetts. This letter and a discussion of it were published in a preliminary report, December, 1914. The letter was particularly helpful to us in formulating in an authoritative way some of the current criticism which was in part the cause of our appointment, criticism ordinarily more apt to be circulated about teachers of mathematics than to be addressed to them.

Reference may also be made in this connection to an address to the Association by Superintendent Morrison of New Hampshire, December 5, 1914 (MATHEMATICS TEACHER ?).

Conferences and correspondence of interest have been had by the committee, or its representatives, with Professor David Eugene Smith, of Teachers College, Professor H. E. Hawkes, chairman of a similar committee of the Middle States Association, Professor P. H. Hanus, of Harvard University, Professor L. L. Conant, formerly of the Massachusetts State Board of Education, and many others.

Co-operative action has been proposed, or suggested, to similar organizations in other parts of the country—to the University Council of Massachusetts, to the American Mathematical Society, and to the recently organized Mathematical Association of America. Some of these proposals have been accepted and others may be expected to yield results of value in due time. Co-operation has also been initiated with the committee of this Association on Mathematics in the Pre-High-School Grades.

Besides this preliminary report, special informal reports of progress have been made at intervening meetings of the Association. Thus Mr. Evans has presented "In Defense of Algebra" (published in *School and Society*, March 18, 1916), and other members have reported on special topics named below. Dean Ferry has also discussed phases of our general problem at a

* It is worthy of note that while three of the eight members live at Worcester, South Hadley and Williamstown, respectively, every member has attended at least six meetings, while the average attendance has been more than six of the eight members. H. W. T.

meeting of the Middle States Association in New York, and at a meeting of the Worcester County Teachers' Association. It has seemed important that our campaign of education should be distributed to a considerable extent both in time and place.

II. Special Studies and Investigations.

Several special studies or investigations have been made by members of the committee, in particular: On the Preparation of Teachers of Mathematics in Massachusetts High Schools, by Dean Ferry; on Records of College Girls in Mathematics, and on Mathematics (or the lack of it) in Massachusetts Normal Schools, by Professor Smith; on the Value of Mathematical Study from the Standpoint of Modern Psychology, by Miss Pierce; on Requirements in Mathematics of Massachusetts High Schools, by Mr. Morse; on the Subject Matter of High School Algebra, by Mr. Evans, with the co-operation of Messrs. Galbraith and Meserve. Two of these special reports—on Preparation of Teachers, and on High School Requirements—are appended to the present report; that on the Value of Mathematics is expected to be published in *School Science and Mathematics*. The results of these special studies are incorporated to a greater or less extent in the body of this report.

The committee has also made much use of published investigations by the International Commission on the Teaching of Mathematics; the American Report, Committees 3 and 4, "Mathematics in the Public and Private Secondary Schools in the United States," by J. C. Brown. See also Appendix 3 on Literature.

III. Discussion.

Present Condition.—Algebra and geometry have long enjoyed a protected, or even monopolistic, position in the high-school curriculum. This position is now challenged: (*a*) by those who object to any prescription whatever; (*b*) by those who object to mathematics, discrediting the value of it from the standpoint of modern psychology and emphasizing the repugnance of students for it, and its limited future utility.

These critical or hostile views gain relative importance from the fact that the determination of educational policies and of

high-school curricula has fallen more or less into the hands of administrative officials entertaining such opinions. The continually increasing competition of newer subjects tends to weaken the position of the traditional subjects. Some teachers of mathematics in the high schools are inadequately prepared for their work, perhaps teaching it only incidentally. Conditions in normal and grammar schools are probably worse. There is unquestionably much room for improvement, and for more economical use of time in mathematical programs and textbooks.

Fundamental Issues.—The committee regards the following questions and considerations as fundamentally important:

1. Is there general public dissatisfaction with the present status of high-school mathematics? If so, to what extent is the criticism well founded and what is the remedy?

2. Is there sufficient reason either for making mathematics a more general requirement for high-school graduation, or for making it more completely optional than now?

3. Is there sufficient reason for asking the colleges to place less emphasis on mathematical requirements for admission?

4. Is the time devoted to mathematics in the schools excessive or deficient, in view of the results attained and the pressure for time?

5. Can better results be accomplished (*a*) by a modified program? (*b*) by a change in the character of textbooks and methods? (*c*) by better preparation of teachers?

Our conclusions may be summarized as follows: That dissatisfaction is not general; that the more radical criticism is not well-founded; that the monopolistic condition is no longer general; that college admission requirements in mathematics are not unreasonable; that most high school teachers in Massachusetts are reasonably well prepared.

On the other hand it will appear that there is need of improvement in the curriculum by eliminating matters too technical or of minor importance, substituting material of greater interest and less difficulty; need of improvement also in the quality of instruction particularly in the seventh and eighth grades.

Public Opinion.—The evidence is, as might be expected, conflicting. Persons taking an iconoclastic position often attract disproportionate attention, and a sufficiently epigrammatic attack

on a tradition associated with past difficulties will gain at least transient sympathy. The committee is not convinced that in public opinion generally there has been any real impairment of the appreciation of mathematics as a fundamental high school subject. It must be remembered of course that in the general democratic evolutoin of high school education, many new curricula have grown up. It need not be insisted that for all of these, mathematics is of the same relative importance. On the other hand, there is real danger that depreciation of mathematics by persons supposed to be expert in matters of secondary education may, unless vigorously met, exert an unfavorable and undue influence on public opinion. It behooves those who believe in high school mathematics to state their case, not trusting merely to the protection of educational inertia. As evidence that there is still substantial appreciation of mathematics on the part of the more intelligent public, reference may be made to an article by Professor Hancock in *School and Society*, June 19, 1915.

The Test of Utility.—The more familiar criticisms of high school mathematics deny the traditional claims as to its value, insist that its content shall be based mainly on utility, and urge the equal or superior value of such recently or partially developed subjects as general science, elementary civics, etc., etc. The argument for utility profits by the increasing public interest in vocational education. The present committee has no direct concern with vocational education as such. We are not unmindful of the long-existing need of the community in this direction, and we gladly recognize the advantage to the community and to the children of bridging the gap between general education and a self-supporting occupation. On the other hand, it would be foolish in an access of enthusiasm for the new to discard what is good in the old. We contend that preparation for life is something larger and finer than preparation for any mere vocation, that there is far more danger under American conditions of too little general education than too much, and that the criterion of utility, narrowly interpreted, is a wholly inadequate test. As a matter of course the boy or girl should devote the invaluable years of adolescence to activities ultimately useful. Such immediately marketable acquisitions as typewriting and stenography make a sufficiently direct appeal to need no

reinforcement with the average boy or girl. But the value and efficiency of life ten or twenty years later will not depend merely upon these minor factors. The youth need not be less fitted for self-support because his education has provided him with broad and inspiring ideas, nor is such fitness the sole end of life. Whatever one's occupation, it will be fundamentally and permanently important to have acquired in youth the habit of exact, orderly and logical thinking, which, if the experience of many centuries of teaching can be trusted, is best acquired by most high-school students in mathematics well taught.

Educational Values.—Our mission is not merely to occupy our pupils' time, nor to make them efficient but unintelligent artisans; it is to educate them to take their place as good citizens of a free community. It seems to us utterly inadequate to propose the substitution of other subjects "severely pursued." The non-mathematical subjects are too different in kind to serve as proper equivalents. A pupil may doubtless work hard in civics, or chemistry, or composition, but none of these can, for the average boy or girl, fully replace geometry or algebra in its emphasis on form or in the complete validity of its logic.

Some of our psychological authorities are fond of asserting that mathematics "does not function" in the life of the child. We can hardly answer this better than in the words of G. St. L. Carson interesting Essays on Mathematical Education.

"The purpose of teaching natural science is to develop in combination the powers of observation and speculation; to train the pupil to use his senses and from the material which they afford him, to frame hypotheses which accord with the material as nearly as may be possible. The purpose of teaching mathematics is to enable him to develop the consequences of these hypotheses, to test their consistence, and to reduce them to the minimum of pure assumption. I say to train him in these things, but it were perhaps better to speak of setting them before him as ideals for which he must strive in his dealings with things as he finds them. A man who has in his mind this chain of processes, observation, speculation, proof of consistence in speculation, rejection of redundant speculation, and finally the erection of deductions on this foundation, is in possession of an intellectual creation which, in beauty alone, is worthy to rank with

the creations of poetry, music, or art; and beyond this, it is a possession which, in so far as it guides his life, will make of him a more efficient laborer and a better citizen."

"The whole world is going through a transformation, due in part to scientific and mechanical inventions and in part to the growth of separate nations, each with its own methods and ideals, of which no man can see the outcome. Our function, the function of all teachers, is to produce men and women competent to appreciate these changes and to take their part in guiding them so far as may be possible. Mathematical thought is one fundamental equipment for this purpose, but mathematical teaching has not hitherto been devoted to it, because the need has but recently arisen. But now that it has arisen and is appreciated, we must meet it or sink, and sink deservedly. Neither the arid formalism of older days nor—I say it in no spirit of disrespect—the workshop reckoning introduced of late will save us. The only hope lies in grasping that inner spirit of mathematics which has in recent years simplified and co-ordinated the whole structure of mathematical thought, and in relating this spirit to the complex entities and laws of modern civilization."

The Disciplinary Value of Mathematics.—The real development of mankind lies in the growth of voluntary attention, which is not passively attracted, but turns actively to that which is important, significant and valuable in itself. No one is born with such a power. It has to be trained and educated. This great function of education is too much neglected. As a reaction against a rigid, empty, mechanical instruction, there swept over the country a wave of electivism which was meant to bring the blessings of freedom, but which did bring primarily a destruction of self-discipline.

The study of mathematics is valuable in helping to form habits of logical memory. We are told that the memory cannot be trained, for there is not one memory, but many; that everything learned is a new memory. On the other hand, we learn that memory is a property of nerve substance, the tendency of a neurone to retain an impression. The neurones in different parts of the brain must obey the same laws; so while there are many specific memories, there are methods which are common to all memory work. Wherever the habits of memory are

formed, they are formed in obedience to these laws and they are consciously formed. The memory habits formed by the study of algebra and geometry are those from which a logical method of memorizing is developed. While the memory is a special memory, its technique is general and this technique should be learned in connection with the best possible material.

Among the habits related to the content of the subject are the habits of economy of thought and of clearness, brevity, and precision in expression. Mathematics is the shorthand language of abstract thought. In learning how to interpret and master its symbols, habits are acquired similar to other language habits, but here the language habits are associated with the symbols which give the greatest economy in thinking and whose use is universal. The world's work requires the constant mastery of symbols, and thinkers are more and more trying to express laws and conclusions as mathematical formulas. "It is only in the least desired occupations that men work entirely with actual things."

While general power is not necessarily gained by the study of mathematics, the student may be so impressed by the perfection of mathematical reasoning, that an ideal is formed which gives a standard by which his thinking in all other subjects is consciously tested. It is generally recognized that "precise thinking is gaining ground over vague theorizing in many scientific and practical activities of the day and elementary mathematics has been and is the model of precise thinking in more difficult fields."*

Various types of thinking are necessary to solve all the problems of life. There is one method for mathematical reasoning, another for the physical science, which differs from that of the biological sciences, while the psychological and sociological sciences have their own special methods of thought. Every high-school pupil should know the rudiments of all these methods; for no one can take the place of the other, and all are needed if we are to make correct judgments and right decisions in all the affairs of daily life.

The habits of orderly accuracy of thought and the methods of persistent concentration in attacking problems have in them

* J. W. A. Young.

elements that are common to habits formed in other ways, and in so far as two subjects have identical elements, the transfer of training from one to the other will be possible. Since mathematics has many applications to other subjects and to life, there are many opportunities for this transfer, but the ability to make it will depend upon the mental attitudes, upon the aims and ideals which have been formed.

Herein lies the greatest value to be derived from the study of elementary algebra and geometry. The ideals of neatness, accuracy and systematic arrangement will influence all other work. All thought expression will be more direct and clear and concise as it follows the models of mathematical thinking. Unconsciously we shall test all our statements by the logical standards of geometry. Careless methods of work will not give satisfaction; that will come only when there is systematic arrangement and careful classification.

These greatest values, like all the greatest things of life, cannot be reduced to exact measurement. A person is bigger than the sum total of all his specific habits by just these immeasurable attitudes and ideals. What the experiments of the psychological laboratory have shown us is the great influence of the mental attitude upon the formation of the various special habits. When there is a definite thing to do with a definite purpose for doing it, the results obtained are far superior to those under other conditions. Just here is where mathematics excels other subjects, for nowhere else can such definite tasks be assigned and such clearly defined ends be given.

Since the formation of these attitudes of mind and ideals of method must be made consciously, much depends upon our insight and ability to present the subject with more inspiration, with a better defined aim, and with a clearer view of its values; so that the transfer of these attitudes and ideals to other subjects will be made possible.

To put the matter more specifically, it may be expected that in the high-school study of algebra pupils can obtain things of fundamental advantage to them in further thinking on any subject, as follows:

1. Direct and careful statement of their own previous knowledge, as to numbers and their relations, and as to measurable

objects. This will in the case of almost every pupil include facts—for example, about inequalities—of which their knowledge was unconscious.

2. Practice in examination of what might be called narrative statements, in the selection of material facts therefrom, and in the expression of those facts in symbolic form. In a great many cases this includes the stripping of irrelevant connotations from a term, and the simplification of the concept denoted by it.

3. The development of an abstract symbolic statement into new forms, and the concrete interpretation of those new forms.

4. The enlargement of the concept of number, the correlation of number with measurable objects, and the idea of continuous number. However vague this idea may be in the pupil's mind, it is expected and generally attained in the study of this early algebra and geometry.

5. Repeated experience of logical argument.

6. Experience in taking the results of one argument as the data of another, the result of the second as the data for a third, and so on.

7. Appreciation of the advantage of technical terms, and of the advantage of a non-redundant definition. The nature of a definition (genus and specific difference).

8. A great deal of practice in exact statement, and in the appreciation of exact statement, of simple and novel facts.

Mathematics as a Test of Aptitude.—The committee is of opinion that a primary purpose of general education is to afford the student a test of his aptitude. It is a fundamental principle of our democratic society that career shall depend upon capacity and aptitude, not merely upon external circumstances. Fitness for important fields of professional activity—engineering and the exact sciences—is determinable to a considerable extent in the high school by aptitude for mathematics.

It seems to us hardly possible to determine such aptitude—or inaptitude—earlier than about the fifteenth year. Every boy who is to graduate in a general high school course should have there at least an introductory course in algebra and geometry, designed largely as a test of the advisability of further study in that field.

College Admission.—College education, however wide its pos-

sible range, should certainly imply a considerable emphasis on that habit of exact, orderly and logical thinking which we have attributed to high school mathematics well taught.

It does not appear that any considerable number of young people who ought to go to college are at present deterred from so doing by entrance requirements in mathematics. It seems to us that it would be entirely unsafe for a college to accept any other present substitute for a moderate requirement in algebra and geometry.

As we are not aware of any present tendency in this direction on the part of colleges, or any demand for it on the part of schools, in New England, more extended discussion of the question seems needless.

The School Curriculum.—The question of time is naturally fundamental in all curricula. If time were abundant, even the critics of mathematics might be mild; with conditions as they are, it is only fair that the teachers of mathematics should use what time they are assigned with the utmost economy and should be ready to meet competitive claims by justificatory argument as to the relative value of particular topics.

European Curricula.—In a recent publication* of the United States Bureau of Education a comparative study is made of the mathematical curricula by years in the United States and foreign countries.

Beginning with the eighth school year—corresponding to the year next preceding our high schools—

"It is customary in all of the European countries to teach algebra and geometry simultaneously. During the eighth school year the time is about evenly divided between these two subjects, from two to three hours a week being devoted to each. An attempt is not made to fuse the subjects, but the interrelations between them are kept constantly in mind, and the pupil is not permitted to forget his geometry while studying his algebra, or vice versa. Each subject is considered an instruction unit, but it is used whenever possible as a tool in the study of the other. By the time a European boy has completed his eighth school year, he is at least a full year in advance of the American boy in his knowledge of mathematics."

* Curricula in Mathematics, Bulletin, 1914, No. 45, by J. C. Brown.

"In all European schools both algebra and geometry are taught during the ninth school year. In most of the countries the time is divided evenly between these two subjects, from two to three hours a week being devoted to each. Here, as in the preceding year, the subjects supplement each other, and no attempt is made to fuse them. The relations between the two subjects are emphasized much more extensively abroad than in the United States. In most of the schools of Europe the distinction between plane and solid geometry is less marked than in the schools of the United States. This is, in part at least, due to the fact that models and drawings are very extensively used abroad. During the ninth school year the work in algebra in most of the European schools is but slightly more advanced than in the best schools of the United States. In some of the schools of Austria, France, Denmark, Holland and Hungary the course is somewhat more advanced than in the United States. In these schools logarithms, proportion, and quadratics are studied."

"The tenth school year is the second year of the secondary school in the United States. In most of the schools the entire year is devoted to the study of plane geometry. In some of the schools half of the year is devoted to the study of algebra and the other half to the study of plane geometry.

"In the schools of Europe the mathematics of the tenth school year is distinctly in advance of that in the United States. The difference in the mathematical courses in the two countries is most marked in the subjects of geometry, trigonometry and drawing."

"The eleventh school year is the third year of the secondary school of the United States. In most of the schools the course in mathematics includes a half-year of algebra and a half-year of solid geometry. In a few of the schools, the first half-year is devoted to the study of solid geometry and the last half to the study of trigonometry."

"When a European boy has completed the eleventh school year, if he has elected the scientific course, he has studied more mathematics than is offered in any except a very few of the most progressive secondary schools in the United States."

"The twelfth school year is the last year of the secondary school in the United States. In many of the schools the first

half of the year is devoted to the study of solid geometry and the last half to trigonometry or to business arithmetic. In some of the schools the first half of the year is devoted to the study of plane trigonometry and the last half to the study of college algebra.

"In practically all of the European countries the twelfth school year begins or ends with a comprehensive review of the mathematics of the preceding years. Differential and integral calculus are offered in the schools of Austria, Belgium, Denmark, France, Sweden, Switzerland, Russia, Germany and Roumania.

"The relations between algebra and geometry are especially emphasized in France and the relations between mathematics and physics receive special emphasis in Germany, Holland and Switzerland.

"When a European boy has completed his twelfth school year he has had the opportunity of studying more mathematics than is offered in any of the secondary schools of the United States. He has had more practice in applying his mathematics in physics, cosmography, and mathematical geography than is the case with the American boy. The simultaneous study of several mathematical subjects results in a more complete mastery of each. He sees the unity of mathematics in a way that is seldom true with the American boy. He can use his arithmetic and algebra in the solution of geometrical problems and his arithmetic and geometry in the solution of algebraic problems much better than the average American boy. He has some knowledge of analytic geometry and of the infinitesimal calculus. The frequent drills and reviews so common in European schools have furnished him with a larger number of mathematical facts and formulas that he can use more readily than his American brother. Mathematics to him is an interesting and a fruitful subject, because he has learned to appreciate something of its deeper significance."

"Abundant provision is made for daily drill in mathematics. The educator of Europe realizes that this daily drill is absolutely necessary in order to give the pupil a real mastery of number facts and relations. A little smattering of the subject will not suffice. The pupil is expected to know thoroughly certain facts and principles and to this end daily drill is provided. No small

part of the thoroughness in detail, which is so characteristic of most of the schools of Europe, may be traced to this drill. The American pupil has some information on a great variety of topics, but much of his knowledge is vague and indefinite, rather than clear-cut notions about definite things.

"Everywhere algebra is introduced earlier than in the United States. In certain of the German schools some work in algebra is introduced during the sixth school year, and in no country, except the United States, is this introductory work posponed later than the seventh school year.

"Some instruction in constructional, observational, or intuitive geometry is always offered during the sixth, seventh and eighth school years. Much emphasis is placed upon estimates and constructions.

"Everywhere the attempt is being made to find genuine applications of mathematics that are really within the experience of the pupil and to link the subject of mathematics as closely as possible with the activities of real life. Drawing and physics are frequently taught by the same teacher, and the correlation between these subjects is found to be to the advantage of each.

"European school men believe that a course in mathematics should be planned by those who know some mathematics rather than by educators who are practically ignorant of the subject. The reports do not indicate that the schools of Europe are hearing a demand for weak algebra and anæmic geometry, or even for no work in these subjects. If any pressure of this sort exists, it has hitherto produced no modification of the course of study."

It might seem out of place for the present committee, as an interested party, to formulate its own views as to the time requirements of mathematics as against other subjects. The cumulative evidence of the experience of other countries is however certainly significant and impressive. It may also fairly be added that there are two important distinctions between mathematics and some of its newer competitors. In the first place it will be not a few decades before they will be worked into thoroughly teachable condition—involving as this does not merely the discovery, correlation and selection of facts, but the preparation of both teacher and textbooks. In the second place, mathe-

matics must be learned in the school or not at all; general science, civics, etc., permeate our modern environment.

Improvement of Present Conditions.—None of the questions so far discussed seem to us comparable in real importance with those as to possible improvement. People who disagree as to value or utility of mathematics are not easily brought even to a common basis for argument.

But there is continual need of improvement and if we neglect any effort in that direction we shall have failed in a vital obligation.

The best possible way to meet this increasing competition of other subjects is to improve our own both as to content and methods. Some of the criticism aimed at us connects itself with conditions long since outgrown in the best schools. The committee, like the Association, must stand for the best in mathematical education.

IV. Conclusions and Recommendations.

High School Requirements.—As shown in Mr. Morse's report, it is no longer generally true that high-school graduation depends on ability in, or the completion of, a substantial mathematical requirement. We, therefore, offer no specific recommendation on this point beyond that embodied under curriculum, below.

College Entrance Requirements in mathematics seem to us not unreasonable in amount for candidates in general. It seems reasonable, and is a fact, that the boy or girl who might well attend college in spite of inability to meet such a requirement has opportunities to do so, though less freely in this than in other parts of the country.

Curriculum.—In view of the evidence in the country at large that above the sixth grade public-school education is likely in the future to be organized on the basis of a junior high school and a senior high school, each three years in length, we have preferred to base the following recommendations on that plan. Such modifications as would be needful to fit present conditions may be inferred. In six-year high schools, the first three grades (junior high school) should include

(*a*) the essentials of arithmetic, omitting the less important denominate numbers, and all intricate matters of commercial arithmetic, which lie outside the possible experience even of teachers. The simple arithmetic of the home, of the farm, of industry, of ordinary banking, of simple investments and similar topics should be emphasized. This should include a very thorough drill in computation, easy methods of approximation, percentage and its varied applications;

(*b*) the elements of algebra as a condensed notation for the processes already studied in arithmetic, with special emphasis on important formulas and the solution of easy equations including quadratics; also the solution of simple problems;

(*c*) in geometry, a considerable amount of informal geometry —constructive or intuitional—with varied problems in mensuration; the beginnings of geometrical demonstration. Statistical graphs and graphs of simple equations should have been used in connection with (*a*) and (*b*).

To this extent mathematics should be a required subject. Enough should have been taken to test aptitude for further similar study, or for college preparation.

In the *senior high school*—tenth, eleventh and twelfth grades —instruction should be offered

(*a*) *in arithmetic,* the more advanced or more specialized vocational topics; the use of logarithms and the slide-rule;

(*b*) *in algebra,* a second course, differing from the earlier one in being more formal and more logical, including a thorough treatment of quadratic equations, due attention to such matters as radicals, exponents, factoring, binominal theorem, etc., and the solution of more difficult problems;

(*c*) *in geometry,* the more important theorems of plane and solid geometry, additional work in mensuration; the trigonometric solution of the right triangle;

(*d*) *in trigonometry,* the theory and solution of the plane triangle, with varied applications.

In (*b*), (*c*) and (*d*) the idea of function—*i. e.,* of related variables—should be introduced; also if possible, some simple illustrations of the derivative—*e. g.,* slope and speed.

How much of this program would be taken by the individual student would depend upon circumstances, but *trigonometry,* on

account of its simplicity and interest, should be generally attractive.

How far the several disciplines should be combined in composite courses seems to us a matter still awaiting experimental determination. We are inclined to favor the general principle of parallel rather than blended courses in algebra and geometry, but some overlapping of each by the other is desirable, as in the use of graphs in algebra and of algebraic methods in geometry.

In *four-year high schools* much will naturally depend on the possibilities of the grammar school preparation, but in our judgment the aims should conform in a general way to the above for corresponding grades. At the end of the first year in the four-year high school, the student should have had a variety of elementary work in both the solution of algebraic equations and in observational geometry and mensuration, with a few simple proofs. In the second year he should be enabled to choose between elementary trigonometry, higher arithmetic and college preparatory work in algebra and geometry.

(*A*) At the end of the first year the student should have cogered substantially the ground indicated as desirable for junior high school work. Omissions, if any are necessary, should be rather of application and practice than of important topics. The committee is of the opinion that to this extent mathematics should be required in the four-year high schools.

(*B*) In the second and succeeding years the pupil should have the opportunity to continue and complete the course of study in mathematics as outlined above for the senior high school.

Preparation of Teachers.—It is the belief of the committee that the preparation of teachers of mathematics in the senior high schools (above the ninth grade) should include the completion of not less than three years of college mathematics, comprising besides the ordinary trigonometry, analytic geometry, calculus with applications, and practice in geometrical drawing work in one or more of the following fields: higher algebra, modern geometry, mechanics, or applied mathematics. In addition, some knowledge of the history and fundamental concepts of mathematics, and some professional training in the teaching of mathematics are desirable.

Teachers of mathematics in the junior high school (seventh,

eighth and ninth grades) should have had instruction in plane and solid geometry, trigonometry, and college algebra (including graphs), or an equivalent with much attention to concrete applications, *i. e.*, the ordinary first-year college mathematics. Preparation to this extent should be regularly provided in the normal schools, as well as instruction in the history and teaching of arithmetic, algebra and geometry.

Procedure.—Recognizing that the changes proposed above are much too far-reaching to be dealt with independently by this Association, the committee presents them as a basis for further study and discussion in co-operation with similar bodies in other parts of the country and with the Mathematical Association of America. It is respectfully recommended that the present committee be discharged and that the council be requested to open negotiations with such other bodies with a view to the organization of a national joint committee, or to such other action as may seem best calculated to secure thorough and adequate consideration of the above recommendations on the part of teachers, administrative officers, and colleges.

G. W. Evans, *Charlestown High School;*
F. C. Ferry, *Williams College;*
A. V. Galbraith, *Middlesex School;*
F. P. Morse, *Revere High School;*
C. D. Meserve, *Newton High School;*
Harriet R. Pierce, *Worcester High School;*
Sophia E. Smith, *Mt. Holyoke College;*
H. W. Tyler, Chairman,
Mass. Institute of Technology.

Appendix I.

An Investigation with Regard to Algebra and Geometry in the High Schools of Massachusetts. By F. P. Morse.

In April, 1915, a questionnaire was sent to the larger high schools of the state outside of Boston. The principals of these schools were asked five questions:

1. Is algebra required for graduation in your school?
2. Is geometry required for graduation?

3. In what years are these subjects offered?

4. How many times per week?

5. Do you think that algebra and geometry should be required for admission to college?

Ninety-seven copies of the questionnaire were sent out and eighty-seven replies received. A summary of the replies is given below.

Is algebra required for graduation? Yes 16, No 65.

Five schools require the subject but allow the pupil to be graduated even if it is not satisfactorily completed. One school requires only sixteen weeks of simple equations and problems. It is evident therefore that in 70 of the 87 schools from which replies were received it is possible for a pupil to receive a diploma without completing a course in algebra.

Forty-seven of the 65 replies which are tabulated under number one indicate that in the schools from which they come there is no requirement in algebra outside of the college course, while the other 18 schools require algebra in several courses, or in most courses.

The replies to question No. 2 give the following results:

Is geometry required for graduation? Yes 8, No 79.

Seventeen of the 79 schools tabulated under *No* require geometry in several courses.

Under question 3, of 81 replies that could be tabulated, a large majority reported that algebra is begun in the first year of high school and geometry in the second. In fact only 3 schools reported that algebra is begun in the second year. Two schools have both subjects in each of the first two years.

The last question brought replies showing a variety of opinions. Fifty-three of those replying said yes. Ten said no. Six did not think these subjects should be required for all courses. Two say they should be required only of those who will make subsequent use of them. Seven would make a difference between boys and girls, either not requiring the subjects at all of the girls, or requiring less than of the boys. Four would require one subject, not both. One reply says: "It all depends on the character of the college and the student's plans." Another: "They seem to me a vestigial survival." Another: "Not necessarily. But anyone thinking of going to college ought to be sure he has brains enough to handle them."

It was not possible to tabulate the replies to the fourth question, but it would appear that 4 or 5 periods per week are most commonly given to the first courses in algebra and geometry.

Appendix II

Preparation of the Teachers of Mathematics in the Secondary Schools of Massachusetts

In a report published by the Kentucky Department of Education in 1909 it is stated that a certain young woman planned to make her entire preparation for the teaching of German in "one of the leading high schools" in Kentucky by devoting to the study of that language six weeks at a summer school. The authority for this statement "insists" that 90 per cent. of the modern language teachers in the American schools "do not perform better work than this lady did in first-year German"; one infers that their preparation is thought to be no more thoroughly established than hers.

It is reported that in 1908 only 32.2 per cent. of the 4,668 secondary school teachers in the State of New York were college graduates.

In that portion of the American Report of the International Commission on the Teaching of Mathematics, prepared by Committee III, page 81, one reads that "we must remember that the majority of those teaching mathematics in our high schools are prepared merely on the side of subject matter, while it is safe to say that in many of our smaller high schools algebra and geometry are taught by those who are not familiar with any mathematics beyond these same subjects." The American Commissioners of the same International Commission make the report of the same Committee III their authority for the statement concerning the secondary schools (page 35 of Bulletin, 1912, No. 14) that "The average newly appointed teacher of mathematics is a college graduate who has had only about one year's work (from 90 to 180 class hours) of mathematics beyond the work of the school in which he teaches."

While one must accept the statements of such authorities as true of the country as a whole, the experience of some of those

having to do with teachers of mathematics in the secondary schools of Massachusetts made them seem inapplicable to this particular state. To determine whether one's favorable impression of the preparation of this body of teachers was warranted appeared an interesting task. This problem looked still more inviting when the State Commissioner of Education in Massachusetts, who could be expected to speak with authority, wrote in his letter of July 20, 1914, to the Committee on the Status of Mathematics in Secondary Schools as follows: "In most small high schools today it will be found that the teacher with least special preparation for his work is usually teaching algebra." The commissioner did not except the schools lying within his jurisdiction and one felt that the statement was intended to apply particularly to them. Inasmuch as it could not be learned that anyone had made a study of the question of the preparation of the teachers of secondary mathematics in Massachusetts as a whole, the task was undertaken.

There were sent in April, 1915, and again in January, 1916, to the principals of the 264 high schools listed by the Massachusetts Board of Education letters with enclosed postal cards of questions. The principals were requested to distribute these cards to their teachers of mathematics. The questions proposed to the teachers in the first questionnaire were as follows:

1. Do you teach mathematics only?
2. If other subjects also, please name them:
3. Are you a college graduate?
4. Please underscore the subjects studied by you in college or under instruction elsewhere: College algebra, solid geometry, trigonometry, analytic geometry, differential calculus, integral calculus.
5. Have you had instruction in any of these since graduation from college?
6. How many years have you taught? How many where you are now?
7. Are you a man or a woman?

The questions contained in the second questionnaire were these:

1. What college degrees have you?
2. Are you a graduate of a normal school?

3. Have you taken courses in education? Pedagogy? Teaching of mathematics? History of mathematics? How many semester courses in these subjects? How highly do you value this work in your own case?
4. What subjects in mathematics have you studied beyond trigonometry, analytic geometry, and calculus?
5. How long have you taught?
6. Do you teach mathematics only?
7. Are you a woman?

Replies to the first questionnaire came from 398 teachers, 240 men and 158 women, representing 245 of the 264 schools; and replies to the second came from 369 teachers representing 217 schools. Although some changes in personnel must have taken place between April, 1915, and January, 1916, the teachers who answered the two questionnaires are in general the same, and the replies have been considered as coming from the same body of teachers in both cases.

Of these 398 teachers, 40 per cent. teach mathematics only, 36 per cent. teach mathematics and one other subject (which in half the cases is science), 19 per cent. teach mathematics and two other subjects, 4 per cent. teach mathematics and three other subjects, four teach mathematics and four other subjects, while one teacher combines mathematics with science, English, French, civics, agriculture, history, and athletics!

93 per cent. of these teachers are college graduates, which leaves only 27 who lack the bachelor's degree. These 27 teachers have in several instances had two or three years of college study; they have taught in the average 23 years each; accordingly they passed the college age before attendance at college became a very common thing. Of those graduated from college, 81 per cent. have the degree of A.B., 14 per cent. the degree of S.B., three per cent. the degree of Ph.B. and two per cent. the degree of Litt.B. 13 per cent. of these college graduates have a second degree, which in eight cases out of nine is the A.M., and in a few instances is an engineering degree. One teacher has three degrees,—S.B., A.M., and M.D.; one has the degree of LL.B.; and none has the degree of Ph.D. or Sc.D.

10 of the 398 teachers state that they have studied in college or under instruction elsewhere none of the subjects listed; 97

per cent. of these 398 teachers of secondary mathematics have studied in college or under instruction elsewhere at least as far as through college algebra, solid geometry, and trigonometry; 71 per cent. of the 398 have studied thus through these subjects and analytic geometry; 54 per cent. continued through differential calculus, and 48 per cent. through integral calculus. Thus it is found that less than three per cent. of all these teachers failed to complete at least a year of college mathematics, while nearly half of them carried the study of the subject through three or more college years. The 10 teachers who have done no college mathematics under instruction average to have taught more than 15 years each. Five of them are college graduates and the five non-graduates average to have taught about 23 years each. Only two of the 10 teach mathematics only, and those two have taught 22 and 26 years respectively.

27 per cent. of this entire number of teachers have had instruction in mathematics since leaving college. This means that more than a quarter of these teachers have interested themselves in their subject to the extent of taking courses in it since their undergraduate days.

The subjects which these teachers have studied beyond trigonometry, analytic geometry, and calculus are of interest. Allowing them to be the judge as to the subjects that should be regarded as lying "beyond" the calculus, and using in general their own characterizations of these subjects, it is found that 29 per cent. of the whole body of teachers have studied such subjects. Of these teachers comprising the 29 per cent., one third have studied mechanics; one fourth have studied projective geometry; one fourth have studied differential equations; 21 per cent. have studied theory of equations; 13 per cent., modern analytic geometry; 9 per cent., descriptive geometry; 8 per cent., theory of functions; 7 per cent., mathematical physics; five per cent., determinants; four per cent., Fourier's series; four per cent., theory of least squares; while some two or three have studied each of eight or nine other mathematical subjects regarded as of advanced character.

While 93 per cent. of these 398 teachers are college graduates, only about 8 per cent. are normal school graduates. Half of the graduates of the normal schools are college graduates also.

The four per cent. who were graduated from the normal school and have no college degree average to have taught 23 years each. Apparently the time is past when the normal school course is regarded as affording sufficient preparation for the teaching of mathematics in a secondary school in Massachusetts.

The replies to the second questionnaire enable one to determine how much of professional training this great body of teachers of mathematics has had through the study of pedagogical subjects. It is found that 57 per cent. have taken courses in education; 44 per cent. have taken courses in pedagogy; 29 per cent. have taken courses in the teaching of mathematics; and 15 per cent. have taken courses in the history of mathematics. The teachers who have taken these courses average to have pursued the study of them to the extent of about three semester courses. 30 per cent. of the teachers who have taken these courses report them to have been of "great" value, 19 per cent. regard them of "some" value, 19 per cent. regard them of "very little" value, four per cent. think them valueless, and 31 per cent. make no reply to the question. One teacher, an A.B. of twenty years of teaching experience, states that these subjects are of "blamed little" worth, but admits that he has not studied them. One "would have felt lost" without this training; another declares that he values them "little as actual assistance in teaching, but highly as a gain in power and confidence." The normal school graduates appear to esteem this work no more highly than do those who have not attended a normal school, although a much larger percentage of the former have taken such courses.

The 398 teachers report an average teaching experience of twelve years. 22 have taught only a single year, 22 have taught two years, 17 three years, 33 four years, 25 five years, 73 from 6 to 9 years, 69 from 10 to 14 years, 57 from 15 to 19 years, 64 from 20 to 29 years, 11 from 30 to 39 years, and four for 40 years or more.

As showing the relative rate of change of positions, the reports show that these 398 teachers average to have remained continuously in their present teaching positions for between six and seven years. 49 of these teachers have occupied their present places continuously more than 15 years, 22 have held

them for more than 20 years, five have done so for more than 30 years, and one has occupied the same chair for 40 years.

Inasmuch as the preparation of those teachers who devote all their time to mathematics only is of special importance in this investigation, their returns have been considered separately with the following results. 160 teachers, of whom 92 are men and 68 women, teach mathematics only. Of these, 94 per cent. are college graduates; and the nine non-graduates, one man and eight women, average to have taught 31 years apiece. Only two of these 160 teachers fail to have studied in college or under instruction elsewhere through trigonometry and those two have taught 22 and 26 years respectively. 81 per cent. of these teachers have studied thus through analytic geometry, 65 per cent. through differential calculus, and 58 per cent. through integral calculus. 32 per cent. of these 160 teachers of mathematics only have had instruction in mathematics since leaving college,—as compared with 27 per cent. of the entire body of 398 teachers. These 160 teachers average to have taught fifteen years and to have held their present positions eight years. Nine of them, with an average of 17 years of teaching experience, have taught in only a single school. In every respect these teachers of a single subject show better reports than do the teachers of two or more subjects.

In order to determine the relative preparation of the teachers of mathematics in the small schools, 81 teachers from 81 of the smallest high schools in the state were selected. Their answers show that 59 of them are men and 22 women; that none of them teaches mathematics only, and that 76, or about 94 per cent., graduated from college; that 79 of the 81 have studied through trigonometry, 62 per cent. through analytic geometry, 48 per cent. through differential calculus, and 42 per cent. through integral calculus; and that 11 per cent. have studied mathematics since leaving college. Even in these smallest high schools it is hard to believe in the face of these facts that "the teacher with least special preparation for his task is usually teaching algebra." These 81 teachers average to have taught only seven years, and to have continued in their present positions only two years. Only four of them have taught as many as 20 years. These are in general the young teachers who make their

beginning in the very small high schools and rapidly advance to better places in larger schools.

In comparing the returns of the 240 men with those of the 158 women, it is found that 38 per cent. of the men and 43 per cent. of the women teach mathematics only, and that 96 per cent. of the men and 87 per cent. of the women are college graduates. 97 per cent. of the men and the same percentage of the women have studied through freshman mathematics, 65 per cent. of the men and 69 per cent. of the women have continued through analytic geometry, and 46 per cent. of the men and 50 per cent. of the women have carried their studies in college through the integral calculus or still further. 24 per cent. of the men and 26 per cent. of the women have had instruction in mathematics since leaving college. In general, then, the women show better preparation for their work than the men. Striking differences between the figures for the men and those for the women appear when the length of the period of teaching is considered. Thus, 9 per cent. of the men and 19 per cent. of the women have taught only one or two years; 10 per cent. of the men and only two per cent. of the women are now engaged in their sixth year of teaching; and 16 per cent. of the men and 24 per cent. of the women have taught more than 20 years. It appears, therefore, that the women who forsake a teacher's life do so ordinarily after only one, two, or three years of it, while the men who give up secondary teaching usually do so after at least ten years of that work have been completed.

The men change their teaching positions more frequently than the women. 55 per cent. of the men and only 43 per cent. of the women have remained in their present positions less than four years. Of the men only 29 per cent. have continued in their present places more than six years, while 48 per cent. of the women have so continued. The percentages of teachers who have taught in only one school and have served more than two years there are four for the men and ten for the women.

It seems to have been established that the statements found in many articles, particularly those of the non-mathematical officers of education, concerning the great lack of preparation of the teachers of mathematics in the secondary schools are not applicable to the state of Massachusetts. When 93 per cent.

of these teachers are college graduates, about half of the entire number have studied through the integral calculus in college, more than a quarter of them have had instruction in mathematics since leaving college, and more than half have had professional training in the teaching of their subject,—the average newly appointed teacher of mathematics in the Massachusetts high schools is worthy of a far better characterization than that which has been quoted above from the American Commissioners. Undoubtedly the State of Massachusetts is far more fortunate in this regard than most of the other states; but may one not hope that improvement is showing itself everywhere and that those who speak only after careful investigation will soon find better things to say of the preparation of the teachers of mathematics in the secondary schools of the entire country!

The ideal preparation, even when as moderately characterized as is the case in the report of the present committee, may be long deferred; but the reports of the 398 Massachusetts teachers of algebra and geometry give ground for hope.

APPENDIX III.

Selected References.

Mathematics in the Public and Private Secondary Schools of the United States. U. S. Bureau of Education, 1911, No. 16.

The Teaching of Elementary Mathematics. D. E. Smith.

Teaching of Geometry. D. E. Smith.

The Teaching of Mathematics in the Elementary and Secondary Schools. J. W. A. Young.

The Teaching of Mathematics in Secondary Schools. Schultze.

Aims and Methods of School Algebra. T. P. Nunn.

Essays on Mathematical Education. G. St. L. Carson.

Teaching of Mathematics. R. E. Manchester.

Memorabilia Mathematica. Moritz.

Mental Discipline and Educational Values. W. H. Heck.

Reorganization of Secondary Education; Preliminary Statements by Chairmen of Committees of the Committee of National Education Association, U. S. Bureau of Education, Bulletin 1913, No. 41. (No mathematics.)

Numerous articles in the MATHEMATICS TEACHER, *The American Mathematical Monthly, School Science and Mathematics, Mathematical Gazette, School Review.* Periodicals presumably familiar or accessible to readers of this report.

Correlation and Disciplinary Values. C. N. Moore. *School and Society.* September 11, 1915.

What Courses of Study Should be Taken by a Boy who is Entering High School. H. Hancock. *School and Society,* June 19, 1915.

MAY, 1916.

NEW BOOKS.

A Practical Algebra for Beginners. By THIRMUTHIS A. BROOKMAN. New York: Charles Scribner's Sons. Pp. xvii + 322.

This algebra contains many interesting departures from traditional methods. It centers the year's work around equations, using linear equations for the first half year and quadratic equations for the second. To apply these equations it introduces chapters on "Similar Right Triangles," "Proportion Underlying Levers," "Belted Pulleys and Gears in Mesh," "Formulas Concerning Bolts and Nuts," and "Direct Variation in Geometric Figures; Inverse Variation in Bicycle Pumps and Locomotives." The author recommends that thirty-six days be spent on these chapters.

In addition to the geometry used in the chapters already mentioned, there is considerable geometric application, especially of area and volume formulas. There is, however, no real attempt to combine the two subjects.

Synthetic division, radicals, and evolution have been relegated to the appendix, which also contains some further applications and excellent tables of square and cube roots. Synthetic division is here because it is not yet generally conceded its place in the subject, but radicals are rightly pronounced by the author to give "slight contribution in comparison with the notative of fractional exponents."

The book approaches the subject with much more of the inductive spirit than is usual in algebras, and it has much that should hold the interest of a student. Like all attempts to make the subject practical by introducing parts of other sciences to which it can be applied, its success or failure can only be demonstrated by trial in various types of schools.

New High School Algebra. By WEBSTER WELLS and WALTER W. HART. Boston: D. C. Heath & Co. Pp. vii + 424. Answers 61.

The first 275 pages are the same as the authors' "First Year Algebra," the remainder completes the usual course in second-year algebra. It carries out logically the plan undertaken in the earlier book, bringing the more difficult parts of such subjects as exponents, radicals and factoring, into the second year. The authors take a very sane position on problems and have written a very teachable book. The criticism it is most likely to meet is that certain parts of the work have had the complications largely removed.

Plane and Solid Geometry. By WEBSTER WELLS and WALTER W. HART. Boston: D. C. Heath & Co. Pp. xii + 467.

This is a text with many good features. It contains an abundance of material, including exercises of the usual type and those meant to show the applications of geometry, as in design. In fact it will meet with objection from some because, although it admits of choice of material, it gives the impression of being long, and of having its pages packed with subject matter. The propositions are first proved in full, then somewhat modified by the omission of references, steps, and in some cases of a large part, or all, of the proof. There are some valuable summaries and an unusually complete index. Constructions are introduced early in the first book and are at once used, both in exercises and in the figures of the following theorems. Indeed, it seems that the construction lines, such as those needed to find a midpoint given in the hypothesis, sometimes needlessly complicate the figures. This book seems on the whole to be the best geometry published in the Wells Series.

Solid Geometry. By WILLIAM BETZ and HARRISON E. WEBB, with the editorial coöperation of PERCEY F. SMITH. Boston: Ginn & Co. Pp. xxii + 178. Price 75 cents.

The book begins with an excellent summary of references to plane geometry used in solid geometry. This is followed by a section of introductory exercises on the regular polyhedrons. It uses the traditional procedure in many respects, the principal variations lying in a little freer assumption in some places, a somewhat different order, and the introduction of modern methods, such as the prismatoid formula and Cavalieri's Theorem. The text is attractive, the figures being well drawn, and the pages open enough not to appear unduly difficult.

Interpolated Six Plane Tables of the Logarithms of Numbers, and the Natural and Logarithmic Functions. Edited by HORACE WILMER MARSH. New York: John Wiley and Sons. Pp. xiv + 155. Price $1.25.

The pages are 6½ by 9½, allowing room for easily read tables. The use of heavy lines that bound each set of logarithms having the same leading figures is an excellent device.

The logarithmic functions are interpolated to seconds, and all the tables are in very usable form. They have been checked by such a large number of students that there seems small probability of errors. The book also contains the usual tables of area, volume, weight, etc.

Constructive Geometry. Prepared under the direction of EARLE RAYMOND HEDRICK. New York: The Macmillan Co. Pp. 76. Price 40 cents.

This is a text in elementary geometric drawing along the lines found successful in England. The book contains both the text, which is largely problems, and blank pages for their solution. It is logically worked out, interesting, and quite comprehensive.

The Avoidance of Fires. By ARLAND D. WEEKS. Boston: D. C. Heath & Co. Pp. 128.

This text-book on fire avoidance is an interesting addition to the list of supplementary books that serve a purpose in themselves as well as correlating with the usual school subjects. It might best, perhaps, have a place in the elementary science course. Typical chapters are: "What Fire Is," "Matches and Fire," "The Danger of Gasoline," "The Use of Gas and Electricity," and "Putting out Fires." The chapter of suggestions and the bibliography are especially valuable features.

Plane Geometry. By CLAUDE IRWIN PALMER and DANIEL POMEROY TAYLOR, and edited by GEORGE WILLIAM MYERS. Chicago: Scott, Foresman & Co. Pp. v + 277.

The editor states that "logic has been intentionally sacrificed to insight whenever it was believed that the general geometric interests of the student would be thereby materially subserved." The text emphasizes applications, dividing such exercises into the usual and more technical kinds by starring the latter. It also prints the foundation theorems in black type to emphasize their importance. There are many variations from the older type of book, and the result has many good features.

Methods of Teaching in High Schools. By SAMUEL CHESTER PARKER. Boston: Ginn & Co. Pp. xxv + 527.

There is little enough usable material on the teaching of high school subjects to make this book especially welcome to secondary school teachers. It has many very valuable chapters, and it contains much helpful material. On the other hand, it shows some "local color."

Teachers of mathematics will be particularly interested in its questioning of the value of the present high school mathematics. The bibliography and frequent references are especially valuable.

On the whole, it is a well-organized book that any secondary school teacher would do well to read.

Blue Bonnet Keeps House. By CAROLINE E. JACOBS and LILA H. RICHARDS. Boston: The Page Company. Pp. 346. $1.50.

In this latest volume of the series we have an interesting account of Blue Bonnet's social life in the new home in the East and her experiences as a senior at Miss North's.

The Girl from the Big Horn Country. By MARY ELLEN CHASE. Boston: The Page Company. Pp. 320. $1.25 net.

This is the story of a young girl who has lived all her life on a western ranch. She comes east for a long visit and finds it hard to adapt herself to the ways and manners of the new life. How she does it makes an entertaining story and one full of life and interesting characters.

Six Star Ranch. By ELEANOR H. PORTER. Boston: The Page Company. Pp. 353. $1.25 net.

This is a very well written story of a summer spent on a Texas ranch by a company of young people from the East. It is a splendid story for young and old.

The Violin Lady. By DAISY RHODES CAMPBELL. Boston: The Page Company. Pp. 322. $1.25 net.

This is a description of the life of Virginia Hammond as a student in Paris, of her great success afterwards as the Violin Lady, and finally of the rival who competes with her beloved violin.

Psychology of High-School Subjects. By CHARLES HUBBARD JUDD. Boston: Ginn & Company. Pp. 515. $—.

Some psychologists have written so much which they have had to modify or abandon on more careful consideration, that one comes to select for their reading those authors who have used more careful methods and exercised better judgment from the beginning. The author of this book is one of the latter type and it is a great pleasure to find such a clear and forceful statement of the present position of the best psychological opinion of the day. Some teachers may feel that he is not altogether at home in parts of his discussion of mathematics, yet it is a surprisingly sound discussion and one that every teacher of mathematics should read. The chapter on "Generalized Experience" is particularly strong and interesting, and the chapter on "Teaching Students to Study" is very suggestive and helpful.

Should Boys and Girls be in School Together?
The Private School: Its Functions and Value in the Community.

These two pamphlets are symposiums on the subjects and may be had for five cents each from the Friends' Select School, Philadelphia, Pa.

Chatterbox. Boston: The Page Company.

This volume for 1915 is full of good things.—There are several very interesting long stories: "The Secret Valley" by Mrs. Hobart Hampton being especially fine. The nature studies and biographies are also fine. There is an unusually interesting article on the Romance of Old Furniture which is well illustrated.

NOTES AND NEWS.

THE 26th meeting of the association was held in Teachers College, Columbia University, New York City, Saturday, May 13, 1916. A slight departure from the usual sectional divisions of the program was made. In the morning two interesting reports were given on the Geometry Committee and the Committee on Status of Mathematics respectively. Following these were reports on A Survey by Districts of Recent Criticism of Mathematics Teaching, given by representatives from all parts of the states covered by our association. These reports proved to be most interesting and instructive.

The afternoon program was prepared and conducted by the New York Section and was on the subject of Graphs. Their uses in engineering, chemistry and business were set forth in a most interesting and convincing manner. Various exhibits of graphic work by members of the New York Section were inspected with considerable interest by those present.

PROFESSOR Em. Mougin, President de l'Association amicule des Lycées de Roanne (France), will be pleased to forward to any mathematical teacher (member of our association) a specimen copy of:

1. New Tables of Logarithms with 5 decimals (English edition): Logs of Numbers from 1 to 10,000; Logs of sin, cos, tgs, cotgs from 0 to 100 grades; logs of sin, cos, tgs, cotgs from 0 to 90 degrees; natural lines (0 to 100 grades); natural lines (0 to 90 degrees); supplement and use of the tables;

2. Mural Table: Logs of Numbers from 1 to 10,000, to be posted in classes of mathematics and physics.

Apply by sending a visiting card with both bottom corners cut off (1 cent only).

Attention.—The Tables of Professor Mougin are: the most complete (numbers—grades—degrees), the cheapest (30 cents), the most accurate (fifth decimal underlined), the smallest (56

pages), the most up-to-date (centesimal division: logs and natural lines). Most complimentary testimonials from well-known scientists such as: Professor Laisant, Ecole Polytechnique, Paris; Fehr, Geneva University; Poincaré, of the French Board of Education; Vessist, Paris University; Father Lefebvre, Louvain University; Fontseré, Barcelona University; Chailan and Nau, Catholic University, Paris; Gasco, Valencia University; Canon Stoffaës, Catholic University, Lille; Ségre, Turin University; Gréard, Paris Academy; Compayré, Lyons Academy, Papelier, Orleans; Mansion, Antwerp, etc.

THE spring meeting of the Philadelphia Section of the Association of Teachers of Mathematics in the Middle States and Maryland was held in the Germantown High School, April 18, 1916.

A brief business meeting was held at which the following officers were elected: President—Dr. Jacob B. Krause, Central High School; Vice-President—Mr. Charles H. Strout, St. Luke's School; Secretary—Miss Ruth Munhall, Germantown High School; Member of Executive Committee—Mr. H. Ross Smith, Southern High School; Representative to the General Association—Dr. George Gailey Chambers, University of Pennsylvania.

The program which followed was one of great interest; the title given was "An Experience Meeting" and seven people gave some of their experiences, which proved most profitable to the rest. The seven speakers were: Mr. Jacoby, Mr. Smith, Miss Depue, Mr. Zeiber, Mr. Brecht, Mr. Moyer, and Mr. Fitch.

IT was suggested that a system for marking papers be adopted so that the pupil could in no way change her paper and, returning it, ask for a higher mark. The system suggested was one of lines, which would tell at a glance how much was subtracted for any mistakes made. Likewise from two experiences related, we found that grave trouble may arise if we permit parents to cause us to change the pupil's marks ever so slightly, for if any change can be made then is it possible, according to one parent, to change any mark, even to bring an absolute zero up to a passing mark. The moral of these two experiences was—take a firm

stand and change not. The last experience related caused quite a heated discussion. A method for factoring a trinomial of the form $ax^2 + bx + c$ was given, which, in the mind of the speaker, lessened the chances of the pupil's going astray. Exception was taken to this, the claim being made that no matter which way it was factored it was a guess problem and so long as one had to guess one might as well do it first as last. Before the meeting could agree as to which way the factoring should be taught the time came for adjournment and the question was left unsettled.

THERE has recently been issued in pamphlet form an extract from the last will and testament of Professor Mittag-Leffler and his wife, this will having been made on March 16, 1916, Professor Mittag-Leffler's seventieth birthday. It seems that by the terms of this will the testators have established a mathematical

institute to bear the name of the donors. This institute will be housed in the villa now occupied by Professor Mittag-Leffler at Djursholm, Stockholm. The object is to establish a foundation which shall assist in the development of pure mathematics in the four Scandinavian countries of Sweden, Denmark, Finland and Norway, but more especially of Sweden.

The terms of the will provide for awarding financial aid to students who give particular promise in the field of pure mathematics, and also to award medals and prizes for noteworthy achievements. So far as possible the prizes will consist of a gold medal, and of sets of the *Acta Mathematica*. These will be bestowed personally at the institute, and will be the occasion of a formal ceremony.

Provision is made for the appointment of a director and a committee, which shall have charge of the foundation. Every

six years or less there will be a formal meeting at which the mathematicians of the four Scandinavian countries will assist.

There has been issued also a sumptuous volume giving a complete catalogue of the magnificent library of Professor Mittag-Leffler, which library will be deposited in the institute and become available for students from all parts of the world. There has hardly appeared anything so sumptuous in the way of a mathematical bibliography, and the volume will be sought for by all mathematical bibliophiles.

The founding of this institute is one of the most noteworthy acts for the encouragement of mathematics that has yet been recorded in the history of the subject.

FROM YEAR TO YEAR.

HENRY VAN DYKE.

Let me but live my life from year to year,
With forward face and unreluctant soul
Not hastening to, nor turning from, the goal;
Not mourning for the things that disappear
In the dim past, nor holding back in fear
From what the future veils, but with a whole
And happy heart, that pays its toll
To Youth and Age, and travels on with cheer.

So let the way wind up the hill or down,
Through rough or smooth, the journey will be joy,
Still seeking what I sought when but a boy,
New friendship, high adventure, and a crown.
I shall grow old, but never lose life's zest,
Because the road's last turn will be the best.

THE MATHEMATICS TEACHER

A MAGAZINE DEVOTED TO THE
INTERESTS OF TEACHERS OF MATHEMATICS

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THE · MAN · WHO · SUCCEEDED WHERE · HERBERT · SPENCER · FAILED

SEVENTY-THREE years ago, in Derby, England, lived a young man who had a new *ideal* in watch-making. Day after day he sat bended over his work table, deep in study, with innumerable sketches before him, seeking to make his dream come true.

He was Herbert Spencer, then an inventor, later one of the world's greatest utilitarian philosophers. His idea, to quote from his autobiography, was "a re-arrangement of the works with a view to greater flatness.

Although his principle—that a practical thin watch could be accomplished only through a re-arrangement of the works — was correct, Herbert Spencer failed. "My model," he writes, "proved to have no superiority; indeed it was a bad one."

Herbert Spencer Philosopher

Another man's ideal

In Cincinnati, Ohio, thirty-three years later, another man conceived the same idea. Like Spencer, he saw that *all* watches were not only too large, but too *thick*.

This man was Dietrich Gruen, a Swiss, a young watch manufacturer who had learned his trade under one Martens of Freiburg, Germany, among the most renowned horologists of his time.

He had founded his business in America with the idea of producing a watch of exceptional merit. For its production his thoughts turned naturally to Switzerland, where from time out of mind the finest watches have been produced.

There he gathered together a group of the finest craftsmen and established his first factory for producing watch movements, importing these and fitting them to their cases in America.

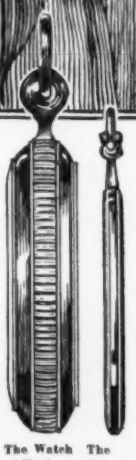
The smallest watch made was what is known as the "18" size. But Dietrich Gruen planned and experimented until he produced the watch known as the "16" size—for many years the popular size watch and the size made to-day by all manufacturers for railroad use. But even this did not satisfy the ideals of Dietrich Gruen, for he realized that the American public wanted a timepiece still smaller and thinner. He saw immediately that no cutting down of existing movements would do. He started then from the bottom, striving to find a new arrangement of movement parts, which, without weakening any individual wheel or pinion, would *build up* a watch of exceeding thinness.

Then came success!

In 1896 he succeeded, through an invention of his son Fritz, to bring out the Verithin Model; but it was not until 1902 (while Herbert Spencer still lived, it is gratifying to state) that Dietrich Gruen brought his thin model to the high standard of accuracy and durability which the name Gruen Verithin stands for.

How he accomplished it is shown by this wheel train illustration below. In the old way the wheels were one above the other like steps. In *his* way he took the smallest wheel and reversed it, placing this small wheel on a line with the larger wheel as shown.

This enabled him to make the Gruen Verithin only *half as thick* as the ordinary watch, without reducing the strength of parts and thus retaining the highest accuracy and durability.



The Watch of Herbert Spencer's time **The Gruen Verithin**

The most beautiful watch in America

The Gruen Verithin has been called "The Most Beautiful Watch in America."

Those who should like to see it so, will find among the best jewelers in every locality one or two who are proud to endorse Dietrich Gruen's masterpiece.

Gruen Verithin Adjusted Models, which are guaranteed to come within railroad time requirements are priced at $25 to $60.

Gruen Verithin Precision Models, which are guaranteed to come within observatory time requirements, recognized by authorities to be the highest timekeepng perfection obtainable, are priced at $50 to $250. The Dietrich Gruen, the world's finest pocket timepiece, $255 to $650.

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